KEY

Transformations and Compositions of Functions

Relations and Functions

Specific Outcome 1: Demonstrate an understanding of operations on and compositions of functions

Sum of functions:

$h(x) = f(x) + g(x)$
$h(x) = (f + g)(x)$

Difference of functions

$h(x) = f(x) - g(x)$
$h(x) = (f - g)(x)$

Domain of the new function h(x) is the domain common to both f(x) and g(x).

To find the graph of the new function: add or subtract the y coordinates at each point.

Examples:

1. Given $f(x) = \sqrt{x+2}$ and $g(x) = \sqrt{-x+2}$,find $f(x) + g(x)$. State the domain and range.

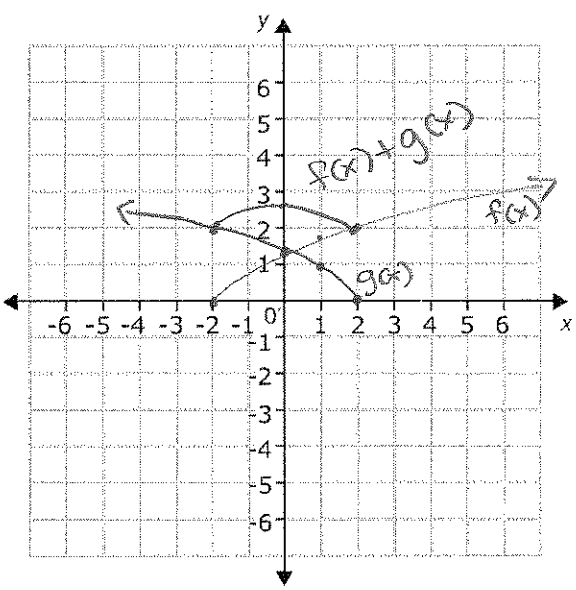


Table of values:

x	$f(x)$	g(x)	$f(x)+g(x)$
-2	0	2	(-2, 2)
0	$\sqrt{2}=1.4$	$\sqrt{2}=1.4$	$(0, 2\sqrt{2})$
1	$\sqrt{3}=1.7$	1	$(1, \sqrt{3}+1)$
2	$\sqrt{4}=2$	0	(2, 2)

(0, 2.8)
(1, 2.7)

Domain: $f(x)$

D: $\{x \mid x \geq -2, x \in \mathbb{R}\}$

Domain $f(x)+g(x)$

D: $\{x \mid -2 \leq x \leq 2, x \in \mathbb{R}\}$

Domain g(x)

D: $\{x \mid x \leq 2, x \in \mathbb{R}\}$

Product and Quotient of functions

Product of functions

$$h(x) = f(x)g(x)$$
$$h(x) = (f \bullet g)(x)$$

Quotient of Functions

$$h(x) = \frac{f(x)}{g(x)}$$
$$h(x) = (\frac{f}{g})(x)$$

Domain for the product of functions is the domain common to both f(x) and g(x). The domain for the quotient of functions also has restrictions for the values of x that make g(x)=0. (Remember: division by zero is undefined)

Examples:

1. Given that $f(x) = 3x - 1$, $g(x) = 7 - x$ and $h(x) = x^2 - 4x$;

a) Determine the value of $(f \bullet g)(2)$

$(f \cdot g)(2) = f(2)g(2)$
$(f \cdot g)(x) = (3x-1)(7-x)$
$(f \cdot g)(x) = 21x - 3x^2 - 7 + x$
$(f \cdot g)(x) = 22x - 3x^2 - 7$

$(f \cdot g)(2) = 22(2) - 3(2)^2 - 7$
$= 25$

b) Determine the value of $\frac{f(3)}{g(-2)}$ $= \frac{3(3)-1}{7-(-2)} = \frac{8}{9}$

c) Determine the expanded expression for $g(x) \cdot h(x)$

$g(x) \cdot h(x)$
$(7-x)(x^2-4x)$
$= 7x^2 - 28x - x^3 + 4x^2$
$= -x^3 + 11x^2 - 28x$

d) Determine the domain and range of the function $\frac{h(x)}{g(x)}$ ← cannot equal zero $x \neq 7$

Domain: $h(x)$ $x \in \mathbb{R}$
$g(x)$ $x \in \mathbb{R}$
$\frac{h(x)}{g(x)}$ $x \neq 7$

Range: $h(x)$ $y \geq -4$
$g(x)$ $y \in \mathbb{R}$
$\frac{h(x)}{g(x)}$

$\frac{x^2 - 4x}{7-x} \neq 0$
$x^2 - 4x \neq 0$
$x(x-4) \neq 0$
$x \neq 0$
$x \neq 4$

Composite functions: Instead of substituting a number into an equation when we compose a function we substitute an equation into another equation.

Notation:

$(f \circ g)(x) = f(g(x))$ Substitute g(x) into f(x)

$(g \circ f)(x) = g(f(x))$ Substitute f(x) into g(x)

Substitute g(x) into g(x) $(g \circ g)(x) = g(g(x))$

Examples:

1. <u>Substitute a number:</u>

Given f(x) find f(3)

$f(x) = x^2 + x + 3$

$f(3) = 3^2 + 3 + 3$

$= 9 + 6$

$= 15$

<u>Composite function: (substitute a function)</u>

Given f(x) and g(x) determine f(g(x))

$f(x) = x^2 + x + 3$

$g(x) = x - 4$

$(x-4)^2 = (x-4)(x-4)$

$= x^2 - 4x - 4x + 16$

$f(g(x)) = (x-4)^2 + (x-4) + 3$

$= x^2 - 8x + 16 + x - 4 + 3$

$\boxed{f(g(x)) = x^2 - 7x + 15}$

2.

If $f(x) = x - 7$ and $g(x) = x^2 - 15$.

a) Find g(f(x))

b) g(g(3))

Solution

a) $(x-7)^2 - 15$

$x^2 - 7x - 7x + 49 - 15$

$g(f(x)) = x^2 - 14x + 34$

b) $g(3) = 3^2 - 15$

$= -6$

$g(-6) = (-6)^2 - 15$

$g(g(3)) = 21$

3. Given that $f(x)=x(x-2)$, determine the equations of:

a) $f(-3x)=$ $(-3x)(-3x-2)$
$= 9x^2+6x$

b) $f\left(\frac{1}{2}x\right)=$ $\left(\frac{1}{2}x\right)\left(\frac{1}{2}x-2\right)$
$=\frac{1}{4}x^2-x$

c) $\frac{1}{4}f(x)=$ $\frac{1}{4}(x(x-2)) = \frac{1}{4}x^2-\frac{1}{2}x$

d) $-2f(x)=$ $-2(x(x-2))$
$=-2x^2+4x$

Practice questions

1. Numerical response: Given the functions $f(x)=7-x$ and $g(x)=3x+1$, Match the graph h(x) that matches each equation. Place your numbers in order ABCD into the numerical response section of your answer sheet. $\boxed{3124}$

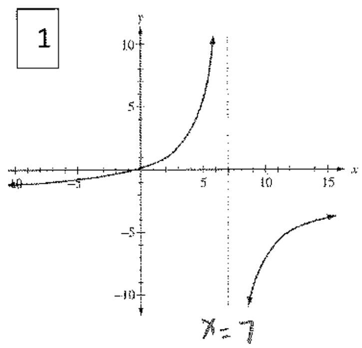
A: $h(x)=f(x)+g(x)$ 3

B: $h(x)=\dfrac{g(x)}{f(x)}$ 1

C: $h(x)=f(x)g(x)$ 2

D: $h(x)=(g\circ f)(x)$ 4

1



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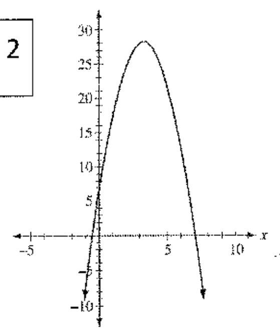

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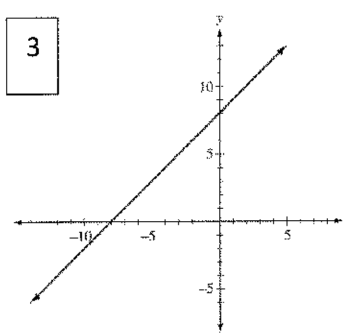

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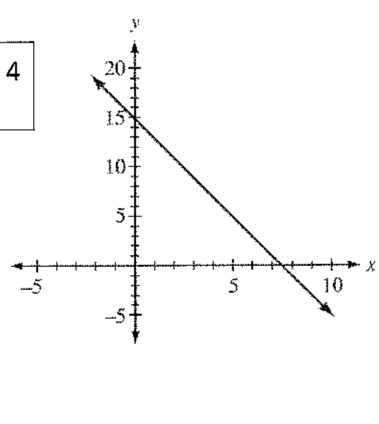


2.

If $f(x) = x^2$ and $g(x) = x + 1$, and $h(x) = f(x) + g(x)$, which of the following is $h(x)$?

A $x^3 + x^2$ **B** $x^2 + x + 1$ **C** x **D** undefined

$h(x) = x^2 + x + 1$

3. Given $f(x) = x^2 + 3$ and $g(x) = 4x + 2$. Find the simplified expression of $h(x) = (f + g)(x)$

$x^2 + 3 + 4x + 2$
$= x^2 + 4x + 5$

a. $h(x) = x^2 + 4x + 5$
b. $h(x) = 4x^3 + 2x^2 + 12x + 6$
c. $h(x) = 16x^2 + 16x + 7$
d. $h(x) = x^3 + 4x^2 + 6x$

4.

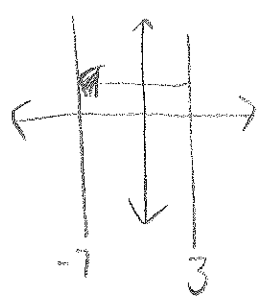
The domain of $f(x)$ is $x \le 3$. If the transformation $g(x) = f(x+10) - 2$ is applied, then the new domain of the function is

10 left → affects x.
2 down → affects y

A. $x \le -10$
B. $x \le -7$
C. $x \ge -10$
D. $x \ge -7$

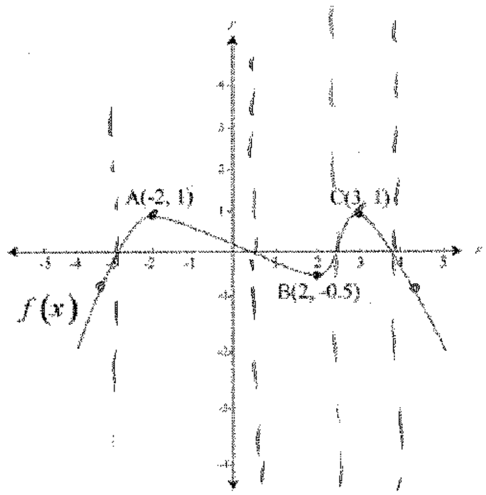
$3 - 10 = -7$

$x \le -7$



Use the following information to answer the next three questions.

Points on the graph of $y = f(x)$ are shown below



$\boxed{5}$ The number of vertical asymptotes found in the graph of $y = \dfrac{1}{f(x)}$ is

A. 0
B. 1
C. 3
D. 4

$\boxed{6}$ The number of invariant points found in the graph of $y = \dfrac{1}{f(x)}$ is

A. 0
B. 1
C. 3
D. 4

$\boxed{7}$ If the graph is transformed to $g(x) = f(2x-4)$, then point **A** becomes $(m, 1)$. The value of m is

$f(2(x-2))$

hor. stretch by $\frac{1}{2}$

2 right

A. 0
B. 1
C. 3
D. 4

$(-2, 1)$

$(-2(\tfrac{1}{2})+2, 1)$

$(-1+2, 1)$

$(1, 1)$

Specific Outcome 2: Demonstrate and understanding of the effects of horizontal and vertical translations on the graphs of functions and their related equations.

Key ideas:

Translating a graph does **not** change the size or shape of the graph. You are simply moving it from one place to another.

Horizontal translation by h units

$y = f(x-h)$

Mapping notation: $(x, y) \to (x+h, y)$

If h is positive, the graph moves right

If h is negative, the graph moves left

Vertical translation by k units (up/down)

$y-k = f(x)$

Mapping notation: $(x, y) \to (x, y+k)$

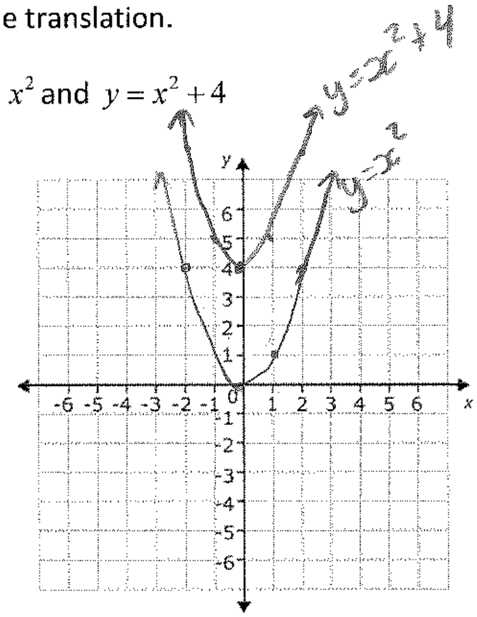
If k is positive, the graph moves up

If k is negative, the graph moves down

Examples:

Sketch the graphs of the following. State the domain and range. Use mapping notation to describe the translation.

1. $y = x^2$ and $y = x^2 + 4$



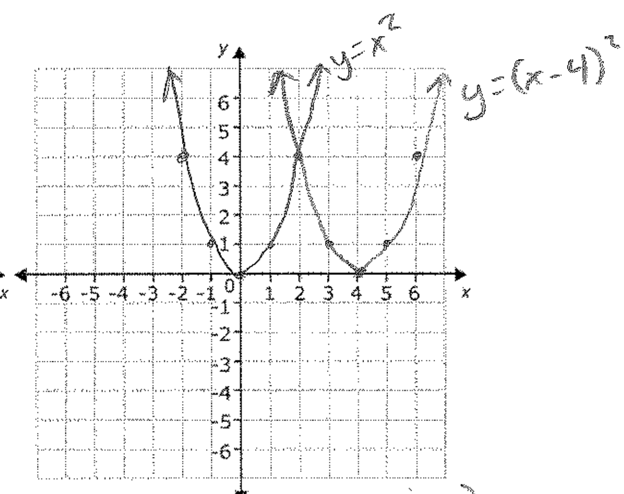
$(x, y) \to (x, y+4)$

both D: $\{x \mid x \in \mathbb{R}\}$

$y=x^2$ R: $\{y \mid y \geq 0, y \in \mathbb{R}\}$

$y=x^2+4$ R: $\{y \mid y \geq 4, y \in \mathbb{R}\}$

2. $y = x^2$ and $y = (x-4)^2$



$(x, y) \to (x+4, y)$

both D: $\{x \mid x \in \mathbb{R}\}$

R: $\{y \mid y \geq 0, y \in \mathbb{R}\}$

2. The point (4, -5) is on the graph of $y = f(x)$. The function is transformed to $y = f(x+5) - 3$. Use mapping notation to describe the transformation and state the coordinates of the corresponding point.

5 left, 3 down

$(x, y) \to (x-5, y-3)$

$(4, -5) \to (4-5, -5-3)$

Corresponding point $(-1, -8)$

3. The graph $y = x^2$ is translated 4 units right and 3 units down. Write the equation of the transformed function.

$y = (x-4)^2 - 3$

4. Describe the transformations that were applied to this equation. $y = 3(4x-2)^2 + 1$

Factor first: $y = 3(4(x - \frac{1}{2}))^2 + 1$

vertical stretch about x axis by a factor of 3.
horizontal stretch about y axis by a factor of 1/4.
horizontal translation $\frac{1}{2}$ unit right.
vertical translation 1 unit up.

Practice:

8. Compared to the graph of $y = \cos(x)$, the horizontal phase shift of $y = \cos(\frac{1}{4}x + \frac{\pi}{2})$ is

$y = \cos(\frac{1}{4}(x + 2\pi))$

a) $\frac{\pi}{8}$ to the left

b) $\frac{\pi}{2}$ to the left

c) 2π to the left

d) 8π to the left

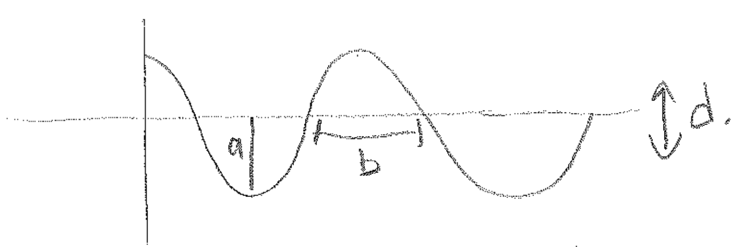
9. The minimum value of the function $f(x) = a\cos(bx) - d$, where $a > 0$, is

~~a) $-b-d$~~

~~b) $-a-d$~~

c) $a-d$

~~d) $a-b-d$~~



10.

$\nearrow$ 1 left

The graph of $f(x) = x^2 - 2$ undergoes the transformation $f(x+1)$.
If a student wishes to graph the transformed function in their calculator, the equation that gives the correct graph is

$(x-1)^2 - 2$

A. $x^2 - 1$
B. $x^2 - 3$
C. $(x+1)^2 - 2$ C is the correct answer
D. $(x-1)^2 - 2$

11.

Numerical Response

A function $f(x)$ is transformed to produce the graph of $g(x) = f(x-7)+8$. If the graph is further transformed by moving it two units left and one unit down, then the new graph can be written as $h(x) = f(x-a)+b$. The numerical values of a and b are, respectively, __5__, and __7__.

$(x-5)+7$

12.

If the graph of $f(x) = x^2$ is transformed to the graph of $y+2 = f(x+1)$, then a true statement regarding the two graphs is

$y = f(x+1) - 2$

$y = (x+1)^2 - 2$

A. The domain, but not the range, is the same.
B. The range, but not the domain, is the same.
C. Both the domain and range are the same
D. The domain and range are both different

Domain is the same
Range different.

Specific Outcome 3: Demonstrate an understanding of the effects of horizontal and vertical stretches on the graphs of functions and their related equations.

Key ideas:

When a graph is stretched, the shape of the transformed graph is not congruent to the original shape.

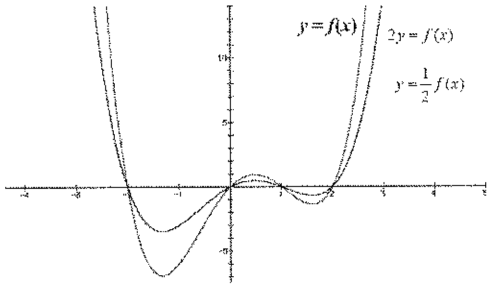
Vertical Stretch:

$y = af(x)$ The function f(x) is vertically stretched about the x axis by a factor of a.

mapping: $(x, y) \rightarrow (x, ay)$ all y coordinates are multiplied by $|a|$.

Invariant points: the x intercepts are invariant points.

if "a" is less than zero, there is also a reflection about the x axis.



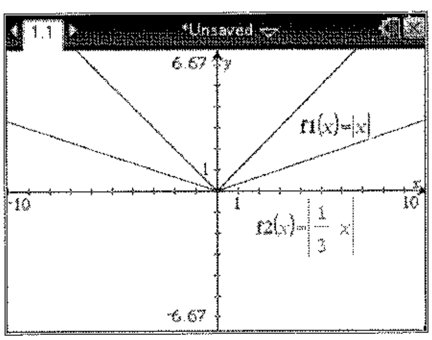
Horizontal Stretch:

$y = f(bx)$ The function f(x) is horizontally stretched about the y axis by a factor of $\frac{1}{|b|}$.

mapping: $(x, y) \rightarrow (\frac{1}{b}x, y)$ all x coordinates are multiplied by $\frac{1}{|b|}$.

Invariant points: the y intercepts are invariant points

If "b" is less than zero, there is also a reflections about the y axis.

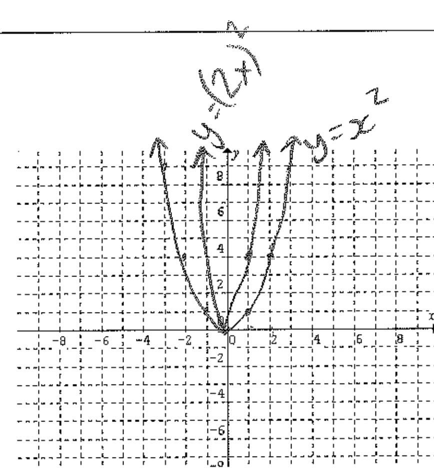


Examples

1. Sketch the graphs of the following equations.

   a. $y = x^2$

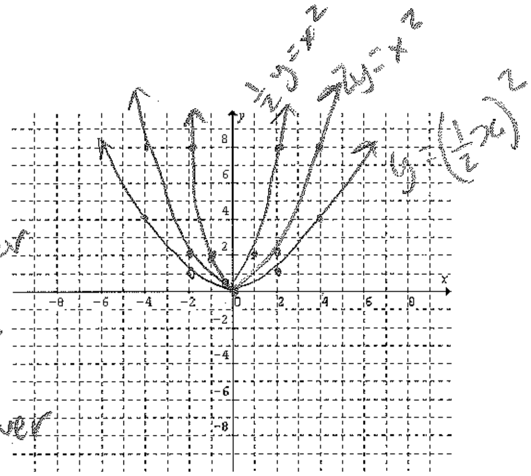
   b. $y = (2x)^2$



c. $y = \left(\frac{1}{2}x\right)^2$ wider

d. $2y = x^2$ wider

e. $\frac{1}{2}y = x^2$ narrower



2. Given that $f(x) = x(x-2)$, state the transformation in words and determine the equations of:

a) $f(-3x) = (-3x)(-3x-2)$

horizontal stretch by $\frac{1}{3}$ and reflect about y axis.

b) $f\left(\frac{1}{2}x\right) = \left(\frac{1}{2}x\right)\left(\frac{1}{2}x-2\right)$

horizontal stretch by a factor of 2.

c) $\frac{1}{4}f(x) = \frac{1}{4}(x(x-2))$

vertical stretch by a factor of $\frac{1}{4}$

d) $-2f(x) = -2(x(x-2))$

vertical stretch by a factor of -2

vertical reflection about the x axis

Practice:

13. The point (2, -4) is on the graph of the function $y = f(x)$. The point which must be on the graph of $y = 2f(-x)$ is

a) (-4,-4)
b) (1, 4)
c) (-2,-8)
d) (4, 4)

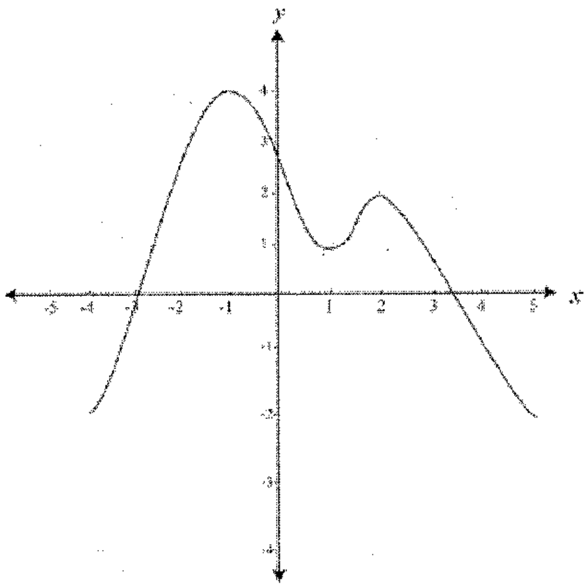
$(x,y) \rightarrow (-x, 2y)$

$(-2, -8)$

14.

Use the following information to answer the next question.

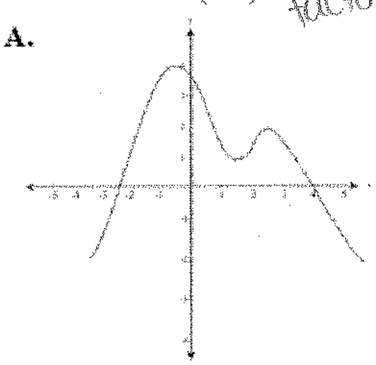
The graph of $y = f(x)$ is shown below



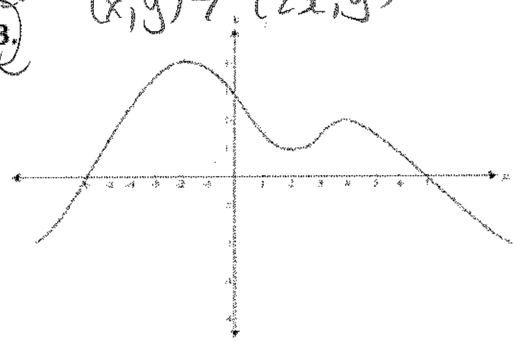
The graph of $f\left(\frac{1}{2}x\right)$ is correctly represented by which of the following?

factor of 2 $(x,y) \to (2x,y)$

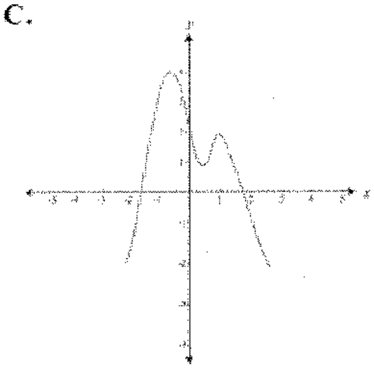
A.



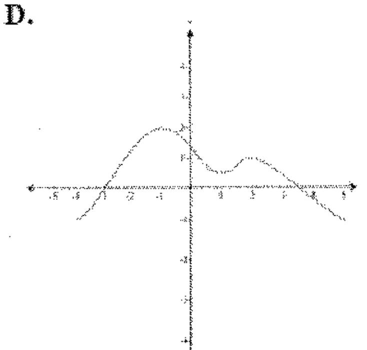
B.



C.



D.



15. How is the graph $y=\left|\frac{x}{3}\right|$ related to the graph $y=|x|$?

a) The graph of $y=|x|$ has been stretched vertically by a factor of 1/3 about the x-axis.

b) The graph of $y=|x|$ has been stretched vertically by a factor of 3 about the x-axis.

c) The graph of $y=|x|$ has been stretched horizontally by a factor of 1/3 about the y-axis.

d) The graph of $y=|x|$ has been stretched horizontally by a factor of 3 about the y-axis.

16. The equation that would cause the graph $y=g(x)$ to stretch vertically about the x-axis by a factor of ¼ and then reflect in the y axis is

a) $y=\frac{1}{4}g(-x)$ $-\frac{1}{4}g(x)$

b) $y=-\frac{1}{4}g(x)$

A is the correct answer

c) $y=4g(-x)$

d) $y=-4g(x)$

17. The graph of $y=f(x)$ undergoes a transformation such that the equation of the transformed graph is $y=f(3x)$. The resulting graph is stretched

A. vertically about the x-axis by a factor of 3 $\frac{1}{3}$ horizontal.

B. vertically about the x-axis by a factor of $\frac{1}{3}$

C. horizontally about the y-axis by a factor of 3

D. horizontally about the y-axis by a factor of $\frac{1}{3}$

Specific outcome 4: Apply translations and stretches to the graphs and equations of functions.

Key ideas:

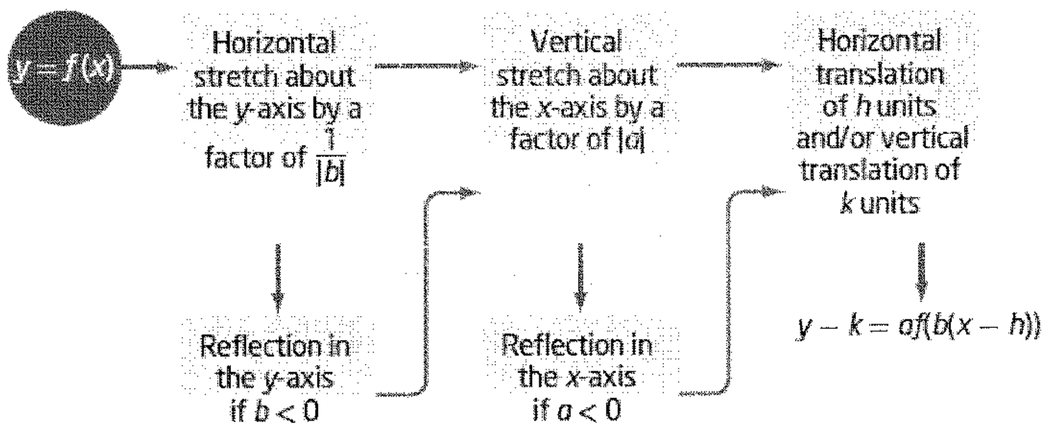
If given an equation always put it in the form $y-k=af(b(x-h))$ first. (A common mistake is to forget to factor b out.) Once you have it in this form

a — represents a vertical stretch by a factor of a, and a reflection in the x-axis if a is negative

b — causes a horizontal stretch by a factor of $\frac{1}{|b|}$, and a reflection about the y-axis if b is negative h causes the graph to move right if positive, and left if negative (horizontal translation)

k causes the graph to move up if positive, and down if negative. (vertical translation)

Figure 4.1: When doing transformations, you need to stretch and reflect before translating. The following flow chart demonstrates the order of transformations. The acronym FRST (fractor, reflections, stretches and translations) may help you remember.



Examples:

1. Place the following equations in the form $y-k=af(b(x-h))$. Then state the parent function and what transformations would need to take place to get the transformed function from the parent function.

    a. $3(y-2)=(x-2)^2$

    $y = \frac{1}{3}(x-2)^2 + 2$

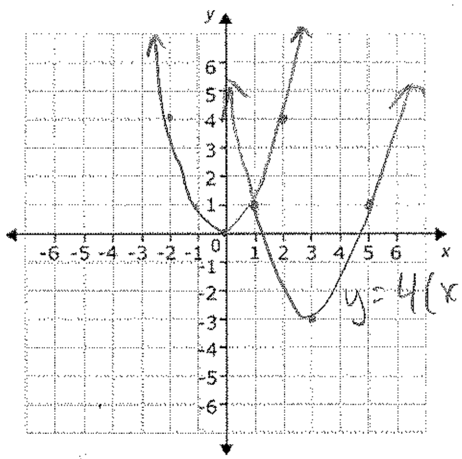
    parent $y=x^2$ is vert. stretched by factor of $\frac{1}{3}$ then it is translated 2 units right and 2 units up.

    b. $f(x)=\sqrt{6x-3}$

    $f(x)=\sqrt{6(x-\frac{1}{2})}$

    parent $f(x)=\sqrt{x}$ is hori. stret. by a factor of $\frac{1}{6}$ and translated 0,5 units right.

2. Graph $y = 4(x-3)^2 - 5$ using transformations of the parent graph $y = x^2$.



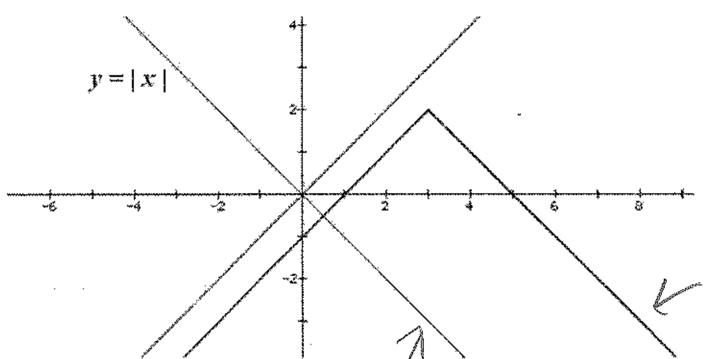
$y = 4(x-3)^2 - 5$

vert. stretch by 4
3 down
3 right.
$(x,y) \to (x+3, 4y-3)$

3. Describe a sequence of transformations required to transform the graph of $y = \sqrt{x}$ to the graph of $y = \sqrt{-\frac{1}{2}(x+8)} + 10$

horizontal stretch by factor of 2
~~horizontal~~ reflection about y axis.
translate 8 units left and 10 units up.

4. Using $y = |x|$ as the parent graph, describe the transformation(s) that occurred to obtain the other two graphs. Write the equation for the transformed graphs.



reflect about x axis, then translate 3 right and 2 up

$y = -|x-3| + 2$

reflect about x axis.

$y = -|x|$

Practice questions:

(-3,6) (-3,-6) (-3,-2)

18. The graph of y = f(x) is reflected in the x-axis, stretched vertically about the x-axis by a factor of ⅓, and stretched horizontally about the y-axis by a factor of 4 to create the graph of y = g(x).

(-12,-2)

For the point (-3, 6) on the graph of y = f(x), the corresponding point on the graph of y = g(x) is
A. (9, 24)
B. (-12, -18)
C. (1, 24)
D. (-12, -2)

19. When compared to the graph of the function $y = x^2$, the graph of the function $y = (x-4)^2$ has undergone a

right 4

a. vertical translation down by 4 units
b. vertical translation up by 4 units
c. horizontal translation right 4 units
d. horizontal translation left 4 units

20.

vert str. 2 5 left.

The graph of $y = -2f(x+5)$ is the same as the graph of

refl x axis

A. The graph of $y = f(x)$ reflected about the x-axis, then shifted five units right, then stretched vertically by a factor of 2 about the x-axis.

B. The graph of $y = f(x)$ reflected about the y-axis, then stretched vertically by a factor of $\frac{1}{2}$ about the x-axis, then shifted five units left.

C. The graph of $y = f(x)$ stretched by a factor of 2 about the y-axis, reflected about the y-axis, then shifted five units left.

D. The graph of $y = f(x)$ stretched by a factor of 2 about the x-axis, reflected about the x-axis, then shifted five units left.

21. If the parent graph is $y = x^2$, the graph of $y = f(-x)+3$ has an equation of

$y=(-x)^2+3$

a. $y = -x^2 + 3$
b. $y = -x^2 - 3$
c. $y = (-x)^2 + 3$
d. $y = (-x)^2 - 3$

22. The graph of $y=\sqrt{x}$ is horizontally stretched by a factor of 3 and translated 4 units to the right. What is the equation of the transformed function.

a. $y=\sqrt{3(x-4)}$

b. $y=3\sqrt{x-4}$

c. $y=\frac{1}{3}\sqrt{x+4}$

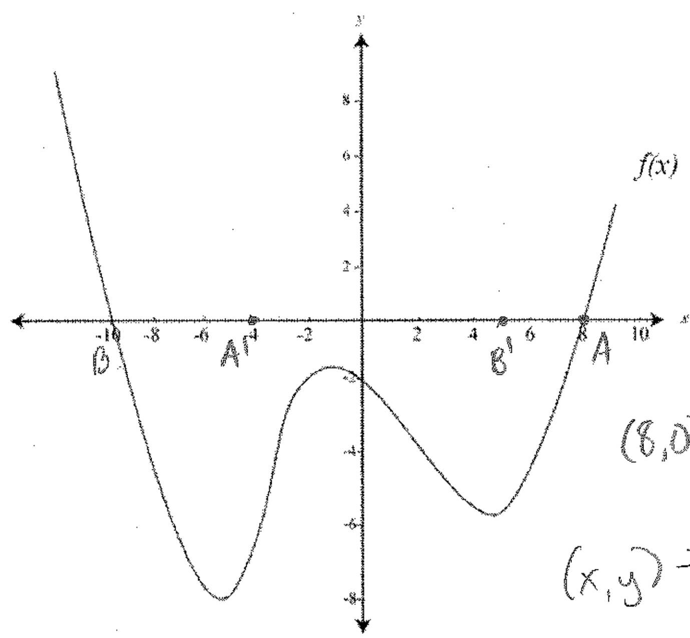
d. $y=\sqrt{\frac{1}{3}(x-4)}$

$\sqrt{\frac{1}{3}(x-4)}$

Use the following information to answer the next 2 questions

23.

The graph of a function is shown below



(8,0)

$(x,y) \rightarrow (-\frac{1}{2}x, y)$

(-4, 0)

(-10,0)

(5,0)

Numerical Response

If the transformation $y=f(-2x)$ is applied, the value of the largest x-intercept is, to the nearest whole number, 5.

24.

The number of invariant points in the graph of $\frac{1}{f(x)}$ is

Not a good question

A. 2
B. 4
C. 6
D. Impossible to determine

---

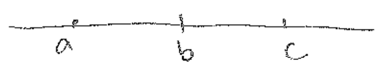
25. The point A(a, b) lies on the graph of $y = f(x)$. If the function $y = f(x)$ was transformed into the function $y = g(x)$, where $g(x) = 2f(x + 1) + 3$, then the coordinates of the transformed point A would be at

$(a, b) \Rightarrow (a+1, 2b+3)$

A. $(a + 1, 2b + 3)$
B. $(a + 1, 2b - 3)$
C. $(a - 1, 2b + 3)$ C is the correct answer
D. $(a - 1, 2b - 3)$

26. A function, $y = n(x)$, has x-intercepts of $(a, 0)$, $(b, 0)$, and $(c, 0)$. Which of the following transformed equations will not affect these values?

A. $y = 2n(x) + 1$
B. $y = -3n(x)$
C. $y = 5n(-x)$
D. $y = -n(x - 4)$



no horizontal or vertical translations, no hor. stretches

27.

The graph of $y = f(x)$ is horizontally stretched by a factor of 3 about the y-axis, reflected in the x-axis, then translated four units right and two units up. The transformed graph is represented by

$-f\ \frac{1}{3}(x-4)+2$

A. $y = -f\left(\frac{1}{3}(x-4)\right)+2$

B. $y = -f(3(x-4))+2$

C. $y = f(-3(x-4))+2$

D. $y = f\left(\frac{1}{3}(-x-4)\right)+2$

Specific outcome 5: Demonstrate an understanding of the effects of reflections on the graphs of functions and their related equations, including reflections through the x axis, y axis and line y = x

Check to make sure the transformed graph is a function. It must pass the vertical line test (have exactly one y value for every x value.) If the transformed graph is not a function you may be asked to restrict the domain to make it a function.

Key ideas:

Reflection about the y-axis: $y = f(-x)$

Mapping: $(x, y) \rightarrow (-x, y)$

Invariant points will be on the y-axis

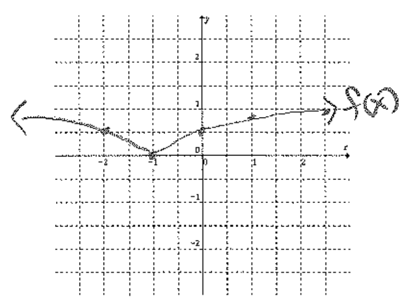
Reflection about the x-axis: $y = -f(x)$

Mapping: $(x, y) \rightarrow (x, -y)$

Invariant points will be on the x-axis

Examples:

1. Sketch the graph of $f(x) = \dfrac{(x+1)^2}{(x+1)^2 + 1}$



Use your graph to sketch the graphs of:

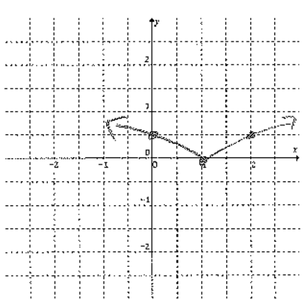
a) $y = f(-x)$

b) $y = -f(x)$

c) $x = f(y)$

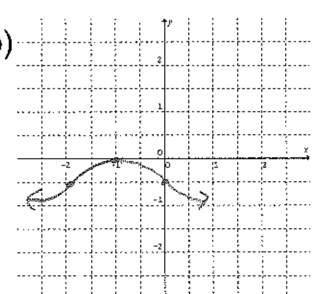
d) Determine which graphs are functions.

a)



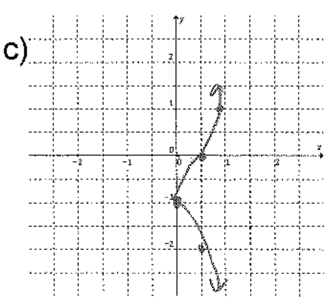
func

b)



func

c)



non func.

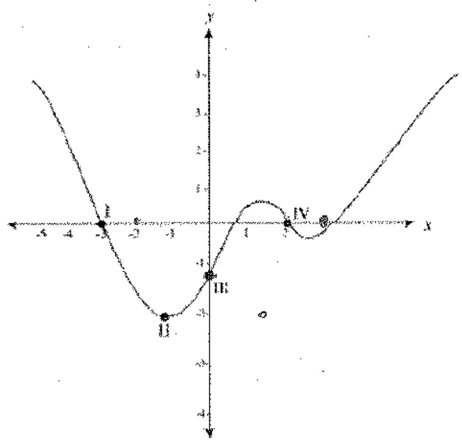
Does not pass vert. line test.

Practice:

28.

Use the following information to answer the next question.

The graph of a function is shown below



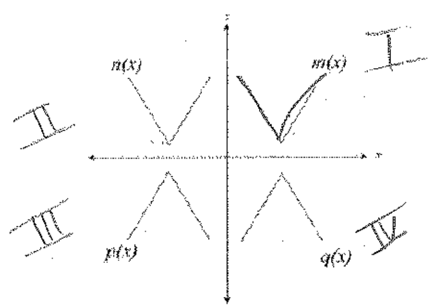
If the reflection $y = f(-x)$ is applied to the graph, the invariant point is

A. I
B. II
C. III
D. IV

29.

Use the following information to answer the next question.

The graph of $m(x)$ is shown, along with three possible reflections.



A student knows the following reflections were used:

1. $y = -f(x)$
2. $y = f(-x)$
3. $y = -f(-x)$

Numerical Response

The reflections used to produce the graphs in quadrants II, III, & IV, respectively, are _2_, _3_, and _1_.

Specific outcome 6: Demonstrate an understanding of inverses of relations.

Key ideas:

If given a graph, you can sketch the inverse by reflecting it through the line $y = x$. If given points, interchange the x and the y coordinates.

Notation: $y = f^{-1}(x)$ or $x = f(y)$

Mapping: $(x, y) \rightarrow (y, x)$

Invariant points will be on the line $y = x$. (where the x coordinate is equal to the y coordinate)

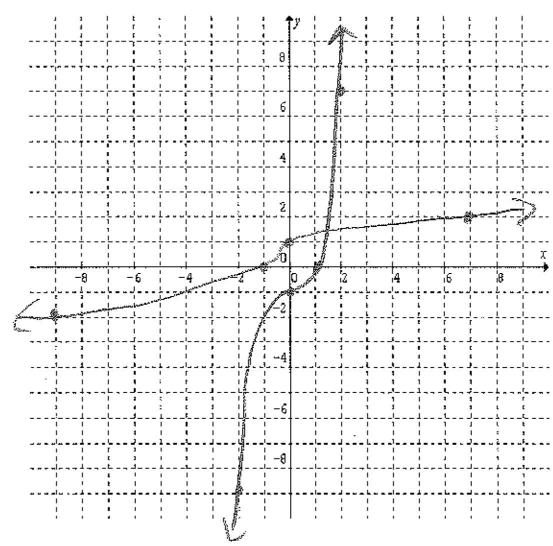
Examples:

1. Graph the function $f(x) = x^3 - 1$.

a) Use this graph to sketch the graph of $x = f(y)$.

b) Use function notation to write the equation of the inverse function.

$x = y^3 - 1$



2. Given the following table of values fill in the table of values for the inverse of f(x)

x	$f(x)$
-3	0
0	3
3	6
6	9

x	$f^{-1}(x)$
0	-3
3	0
6	3
9	6

interchange x's and y's.

3. Given the function $f(x)=\frac{1}{2}x-5$, state the equation of the inverse, the domain and range of the inverse, whether or not the inverse is a function, and the coordinates of any invariant points.

$x=\frac{1}{2}y-5$

$2(x+5)=y$

$\boxed{y=2x+10}$

D: $\{x \mid x \in \mathbb{R}\}$
R: $\{y \mid y \in \mathbb{R}\}$
yes it's a function.
invariant pts where $x=y$.
$(-10,-10)$

4. Given the function $f(x)=x^2+9$, state the equation of the inverse. Is the inverse a function? If it is not place restrictions on the domain so that it will also be a function.

$x=y^2+9$

$y=\pm\sqrt{x-9}$

$y=\sqrt{x-9}$, when $y>0$

$y=\sqrt{x-9}$, when $y<0$

Practice:

30. The inverse of the relation $y=\sqrt{x-8}$ is

a) $y=(x-8)^2$

b) $y=\frac{1}{\sqrt{x-8}}$

c) $x=\sqrt{y+8}$

d) $x=\sqrt{y-8}$

$x=\sqrt{y-8}$

$x^2+8=y$

31.

The graph of $y=(x+1)^2$ undergoes the transformation $y=f^{-1}(x)$.
A true statement regarding the transformed graph is

A. The transformed graph is the ~~reciprocal~~ inverse of the original X
B. The transformed graph is not a function ✓
C. The transformed graph has the same domain and range as the original graph X
D. The vertex of the parabola is invariant X

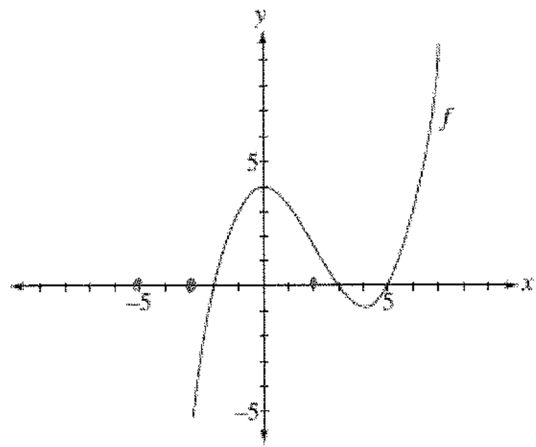
$x=(y+1)^2$

$\pm\sqrt{x}-1=y$

32.

Use the following information to answer the next question.

The partial graph of $y = f(x)$, with all its x-intercepts, is shown below.



The x-intercepts and the y-intercept are integral values.

The transformed function $y = f(-x)$ has x-intercepts of ___i___ and a y-intercept of ___ii___. 4 -5, -3, 2

The statement above is completed by the information in row

Row	*i*	*ii*
A.	$-5, -3$, and 2	4
B.	$-5, -3$, and 2	-4
C.	-2, 3, and 5	4
D.	-2, 3, and 5	-4

33.

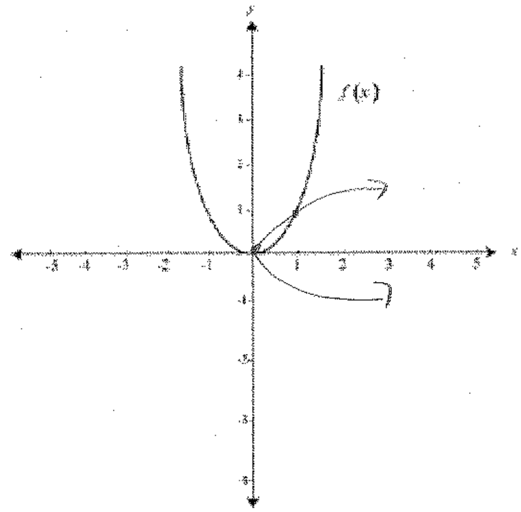
A transformation is applied to the graph of $y = f(x)$ such that the point (2, 2) is invariant. A transformation that can produce this result is

A. $y = 2f(x)$
B. $y = -f(x)$
C. $y = \dfrac{1}{f(x)}$
D. $y = f^{-1}(x)$

34.

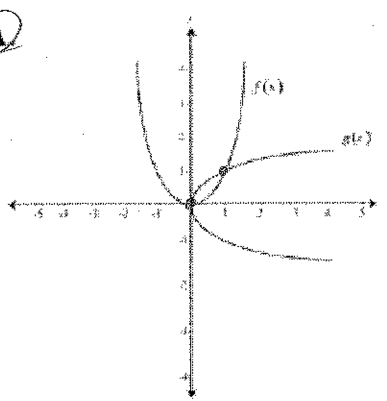
Use the following information to answer the next question.

The graph of $y = f(x)$ is shown below

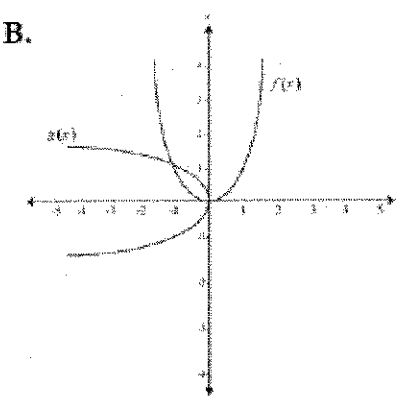


The graph of $f(x)$ and the graph of $g(x) = f^{-1}(x)$ are correctly represented by which of the following pairs of graphs?

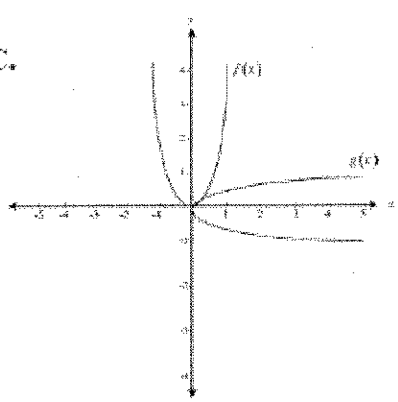
A.



B.



C.



D.