Pre-session Booklet

Specific Outcomes, Achievement Indicators and Student Examples

Math 30-1

R^3

(Revisit, Review and Revive)

Mathematics 30–1 Formula Sheet

For $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Relations and Functions

Graphing Calculator Window Format

x: $[x_{\min}, x_{\max}, x_{\text{scl}}]$

y: $[y_{\min}, y_{\max}, y_{\text{scl}}]$

Laws of Logarithms

$$\log_b(M \times N) = \log_b M + \log_b N$$

$$\log_b\left(\frac{M}{N}\right) = \log_b M - \log_b N$$

$$\log_b(M^n) = n\log_b M$$

$$\log_b c = \frac{\log_a c}{\log_a b}$$

Growth/Decay Formula

$$y = ab^{\frac{t}{p}}$$

General Form of a Transformed Function

$$y = af\left[b(x - h)\right] + k$$

Permutations, Combinations, and the Binomial Theorem

$n! = n(n-1)(n-2)\ldots3 \times 2 \times 1$,
where $n \in N$ and $0! = 1$

$${}_nP_r = \frac{n!}{(n-r)!}$$

$${}_nC_r = \frac{n!}{(n-r)!\,r!} \qquad {}_nC_r = \binom{n}{r}$$

In the expansion of $(x + y)^n$,
the general term is $t_{k+1} = {}_nC_k x^{n-k} y^k$.

Trigonometry

$$\theta = \frac{a}{r}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta}$$

$$\cot\theta = \frac{1}{\tan\theta}$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$

$$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$

$$\sin(2\alpha) = 2\sin\alpha\cos\alpha$$

$$\cos(2\alpha) = \cos^2\alpha - \sin^2\alpha$$

$$\cos(2\alpha) = 2\cos^2\alpha - 1$$

$$\cos(2\alpha) = 1 - 2\sin^2\alpha$$

$$\tan(2\alpha) = \frac{2\tan\alpha}{1 - \tan^2\alpha}$$

$$y = a\sin\left[b(x - c)\right] + d$$

$$y = a\cos\left[b(x - c)\right] + d$$

These are common symptoms associated with test-writing anxiety that you can team to manage effectively. Try some of the following strategies to calm yourself and help you to focus during exam time.

Before the Exam

- Accept that feeling anxious prior to a test is a normal feeling.
- Associate with positive, focused classmates.
- Take some time to relax before you go to bed the night before your test.
- Arrive slightly before the exam prepared with the necessary pens, pencils, calculators or reference materials (It they are allowed).
- Visualize your successful completion of the test.
- Use positive se ·talk such as 'I can do this'.

During the Exam

- Write anything you know or want to remember on the back of your exam such as formulas, steps in a process, or dates in a timeline. It serves as a reminder that you do know something.
- Begin with a question to which you know the answer to build your self-confidence.
- Tense and relax muscles where you feel tension. For example, raise/lower/ roll your shoulders, roll your head from side to side, or flex your fingers.
- Concentrate on an object or spot in the room for a short period of time.
- Focus on your breathing, slowly Inhaling through your nose and exhaling through your mouth.
- Use positive self-talk such as 'I am doing a good job'; 'When I work/read carefully I can figure this out', etc.

MATH 30-1 R^3 Test Tips

1. Start your exam by taking a few deep breaths to calm yourself. If you are feeling anxious, use positive self-talk such as, 'I can do this'," I'm ready for this', or 'This is going to be my best test result'.

2. Immediately write down any Information that you do not want to forget. For example, on a math exam, write down the formulas you may need to know and use for the exam before you begin to look at the exam instructions and questions.

3. Look over the entire test paper or booklet to see what type(s) of and how many questions there are in total. This gives you an overview of the specific content and concepts that are being assessed, the value or number at marks for each question, and the question format(s) to which need to respond.

4. Budget your time for each section of the exam before you begin answering any part of the test. You do not want to spend a lot of time on a portion of the exam that is worth only a small percentage of the total number of marks.

5. Read all of the Instructions carefully before you begin answering any questions. It Is important that you know what you are expected to do and what specifically is required in your response.

s. Start with a question for which you know the answer, you do not have to start at the beginning of the test. Knowing the answer to the first question you try builds your confidence and sets a positive tone for the rest of the exam.

7. Look for clues and information in one part of the test that helps you to answer questions in another part. Useful information is often included in diagrams, charts, graphs, photos, and source information such as the year a photo was

taken or where the author Jived. Pictures, charts, graphs, and diagrams may provide a context for the question you are expected to answer.

8. Answer every question even if you have to guess. Leaving a question blank is guaranteed to earn you zero marks. A guess in a multiple choice question, or working a math question through as far as you are able, gives you an opportunity to 'luck out or earn partial marks.

9. Read over all of your answers before you hand in your test. You want to be sure you've answered all the questions, that your response satisfies what was asked, and that your calculations are accurate. Where necessary include the units of measurement.

10. Finish your answers to written response or essay questions in point form If you are running out of time. This shows the marker that you have understood the question and have an organized and thoughtful response- just not enough time for complete sentences.

11. Stay In the exam room for the entire amount of time. If you finish before the time is up, review your answers. Take a short 'mental break' by rolling your head from side to side and/or shrugging your shoulders. In thinking about the overall test or reviewing specific questions, you may remember additional information to add to some of your responses.

TEST RESPONSE STRATEGIES

There are many strategies to help you maximize your performance on quizzes, unit tests, and final exams. The following provides information on three common strategies that you may find useful.

Five-Pass Strategy- has five steps or 'passes' for answering exam questions:

- *Browsing Pass*- scan the entire exam; note question formats, marks for each section, instructions
- *First Answering Pass*- answer questions you know the answer to and can complete quickly; maintain a brisk pace
- *Second Answering Pass*- answer questions left during ti·1e first pass that require more effort per mark; work the questions through until you experience an impasse or it is taking too long; maintain a steady pace
- *Third Answering Pass*- complete all partial answers; guess at any true/false and multiple choice questions you have left and attempt any questions still unanswered; all questions should now have full or partial answers
- *Review Pass*- check all answers and calculations as time permits

Three-Pass Strategy – has three steps or 'passes' for answering exam questions; is an abbreviated version of the five pass method:

- *Overview*- scan the entire exam; note question formats, marks tor each section, instructions, and questions you can complete easily and quickly
- *Second Pass*- answer all questions you can complete without too much difficulty stopping when the question becomes too challenging or is taking too much time
- *Last Pass*- answer any questions that are left; guess and provide partial answers if necessary

Problem-solving Strategy- has four steps to work through in answering a problem; the problem can be a word problem or a calculation problem:

- *Understand the problem*- What is the question? Do I have the information I need?
- *Develop a Plan*- Have I done something similar before? Can I break the question down into smaller parts?
- *Carry out the Plan*- Is my process logical? Have I shown all of the steps in the process? Have I included the correct units of measurement?
- *Look Back*- Have I answered the question? Does my answer make sense?

MANAGING TEST ANXIETY

Feeling somewhat anxious or 'stressed' before an exam is quite normal for most students. At times the tension helps you to 'rise to the challenge'. Do you feel some or all of the following prior to writing a test?

- Nervousness/Nausea
- Difficulty sleeping the night before
- 'Butterflies' in your stomach
- 'Blanking' or 'not being able to remember anything'

Math 30-1

R^3

Revisit, Review and Revive

This review is set up with six 2 hour sessions.

Session 1 - Relations and Functions/Transformations

a.) Topic 2 - Specific Outcome 1-6
b.) Absolute Value Workbook - Unit 1 and 2

Session 2 - Exponential/Logarithmic Functions and Polynomial Functions

a.) Topic 2 - Specific Outcome 7-12
b.) Absolute Value Workbook - Unit 3 and 4

Session 3 - Rational and Radical Equations

a.)Topic 2 - Specific Outcome 13- 14
b.)Absolute Value Workbook - Unit 5 and 7

Session 4 - Permutations and Combinations

a.)Topic 3 - Specific Outcome 1-4
b.)Absolute Value Workbook - Unit 6

Session 5 - Trigonometry Functions and Graphs

a.)Topic 1 - Specific Outcome 1-4
b.) Absolute Value Workbook - Unit 7

Session 6 - Trigonometry Equations and Identities

a.)Topic 1 - Specific Outcome 5 and 6
b.)Absolute Value Workbook - Unit 8

To make this a successful review please follow the instructions below before you attend your first session.

1. Revisit your notes from the year. Read through your notes, assignments and exams from this course to refresh your memory of what was covered throughout the course.
2. Visit the Quest A+ website and do the practice exam (https://questaplus.alberta.ca/) Click on "Practice Tests" then "Grade 12", then scroll to find "Mathematics 30-1", then click on "Year End Practice" and finally click on "Take the Practice Test"
3. While going through this exam keep your "Pre-Session Booklet" by your side. When you run into a question that you need some help with record it by the appropriate Curricular Outcome/Achievement Indicator. There is a space provided for specific examples that you would like answered during the review.
4. Take the time to visit some other sites that may help with your Math 30-1 Review. Here are some sites that may be useful.
   a. Salisbury High Math 30-1 Page
      (http://www.salcomp.ca/eteachers.php?teacher=689&page=8636)
   b. Math 30-1 Explained
      (http://www.math30.ca/index.php)
5. Come to the review sessions prepared to ask questions and work collaboratively with your teacher and fellow students to make it a positive experience for everyone involved.
6. Make use of the attached Course Year Plan and Diploma Study guide. Plan review early and stick to your schedule.

Examination Specifications and Design

Each Mathematics 30–1 Diploma Examination is designed to reflect the core content outlined in the *Mathematics 30–1 Program of Studies*. The examination is limited to those expectations that can be measured by a machine-scored paper-and-pencil test. Therefore, the percentage weightings will not necessarily match the percentage of class time devoted to each unit. The allotted time for the Mathematics 30–1 Diploma Examination will be **two and a half hours**; however, an **additional half hour** is allowed for students to complete the exam.

Specifications

The format and content of the Mathematics 30–1 Diploma Examinations in the 2012–2013 school year are as follows:

Question Format	*Number of Questions*	*Percentage Emphasis*
Multiple Choice	28	70%
Numerical Response	12	30%

Mathematical Understanding	*Emphasis*
Conceptual	34%
Procedural	30%
Problem Solving	36%

Diploma Exam Content	*Percentage Emphasis*
Relations and Functions	55%
Trigonometry	29%
Permutations, Combinations, and Binomial Theorem	16%

SESSION 1

Topic 2: Relations and Functions

General Outcome: Develop algebraic and graphical reasoning through the study of relations.

Specific Outcome 1: Demonstrate an understanding of operations on, and compositions of, functions.

Achievement Indicator:	Example:
1.1 Sketch the graph of a function that is the sum, difference, product or quotient of two functions, given their graphs.	
1.2 Determine the domain and range of a function that is the sum, difference, product or quotient of two functions.	
1.3 Write the equation of a function that is the sum, difference, product or quotient of two or more functions, given their equations. Write a function *h(x)* as the sum, difference, product or quotient of two or more functions.	
1.4 Determine the value of the composition of functions when evaluated at a point, including: • $f\left(f\left(a\right)\right)$ • $f\left(g\left(a\right)\right)$ • $g\left(f\left(a\right)\right)$	

1.5 Determine, given the equations of two functions *f(x)* and *g(x)*, the equation of the composite function: • $f(f(x))$ • $f(g(x))$ • $g(f(x))$ and explain any restrictions.	
1.6 Sketch, given the equations of two functions *f*(*x*) and *g*(*x*), the graph of the composite function: • $f(f(x))$ • $f(g(x))$ • $g(f(x))$	
1.7 Write a function *h(x)* as the composition of two or more functions.	
1.8 Write a function *h(x)* by combining two or more functions through operations on, and compositions of, functions.	

Specific Outcome 2: Demonstrate an understanding of the effects of horizontal and vertical translations on the graphs of functions and their related equations.

Achievement Indicator:	**Example:**
2.1 Compare the graphs of a set of functions of the form $y - k = f(x)$ to the graph of $y = f(x)$, and generalize, using inductive reasoning, a rule about the effect of *k*.	
2.2 Compare the graphs of a set of functions of the form $y = f(x - h)$ to the graph of $y = f(x)$, and generalize, using inductive reasoning, a rule about the effect of *h*.	
2.3 Compare the graphs of a set of functions of the form $y - k = f(x - h)$ to the graph of $y = f(x)$, and generalize, using inductive reasoning, a rule about the effects of *h* and *k*.	
2.4 Sketch the graph of $y - k = f(x)$, $y = f(x - h)$ or $y - k = f(x - h)$ for given values of *h* and *k*, given a sketch of the function $y = f(x)$, where the equation of $y = f(x)$ is not given.	

2.5 Write the equation of a function whose graph is a vertical and/or horizontal translation of the graph of the function $y = f(x)$.	
Specific Outcome 3: Demonstrate an understanding of the effects of horizontal and vertical stretches on the graphs of functions and their related equations.	
Achievement Indicator:	**Example:**
3.1Compare the graphs of a set of functions of the form $y=af(x)$ to the graph of $y = f(x)$, and generalize, using inductive reasoning, a rule about the effect of *a*.	
3.2 Compare the graphs of a set of functions of the form $y=f(bx)$ to the graph of $y = f(x)$, and generalize, using inductive reasoning, a rule about the effect of *b*.	
3.3 Compare the graphs of a set of functions of the form $y=af(bx)$ to the graph of $y = f(x)$, and generalize, using inductive reasoning, a rule about the effects of *a* and *b*.	
3.4 Sketch the graph of $y=af(x)$, $y=f(bx)$ or $y=af(bx)$ for given values of *a* and *b*, given a sketch of the function $y = f(x)$, where the equation of $y = f(x)$ is not given.	

3.5 Write the equation of a function, given its graph which is a vertical and/or horizontal stretch of the graph of the function $y = f(x)$.	
Specific Outcome 4: Apply translations and stretches to the graphs and equations of functions.	
Achievement Indicator:	**Example:**
4.1Sketch the graph of the function $y-k=af(b(x-h))$ for given values of *a*, *b*, *h* and *k*, given the graph of the function $y= f(x)$, where the equation of $y = f(x)$ is not given.	
4.2 Write the equation of a function, given its graph which is a translation and/or stretch of the graph of the function $y = f(x)$.	
Specific Outcome 5: Demonstrate an understanding of the effects of reflections on the graphs of functions and their related equations, including reflections through the: · *x*-axis · *y*-axis · line $y = x$.	
Achievement Indicator:	**Example:**
5.1Generalize the relationship between the coordinates of an ordered pair and the coordinates of the corresponding ordered pair that results from a reflection through the *x*-axis, the *y*-axis or the line $y = x$.	

5.2Sketch the reflection of the graph of a function $y = f(x)$ through the *x*-axis, the *y*-axis or the line $y = x$, given the graph of the function $y = f(x)$, where the equation of $y = f(x)$ is not given.	
5.3Generalize, using inductive reasoning, and explain rules for the reflection of the graph of the function$y=f(x)$through the *x*-axis, the *y*-axis or the line $y = x$.	
5.4Sketch the graphs of the functions $y = -f(x)$, $y = f(-x)$ and $x = -f(y)$, given the graph of the function $y = f(x)$, where the equation of $y = f(x)$ is not given.	
5.5Write the equation of a function, given its graph which is a reflection of the graph of the function $y = f(x)$ through the *x*-axis, the *y*-axis or the line $y = x$.	
Specific Outcome 6: Demonstrate an understanding of inverses of relations.	
Achievement Indicator:	**Example:**
6.1Explain how the graph of the line $y = x$ can be used to sketch the inverse of a relation.	

6.2Explain how the transformation (*x*, *y*) => (*y*, *x*) can be used to sketch the inverse of a relation.	
6.3Sketch the graph of the inverse relation, given the graph of a relation.	
6.4Determine if a relation and its inverse are functions.	
6.5Determine restrictions on the domain of a function in order for its inverse to be a function.	
6.6Determine the equation and sketch the graph of the inverse relation, given the equation of a linear or quadratic relation.	
6.7Explain the relationship between the domains and ranges of a relation and its inverse.	
6.8Determine, algebraically or graphically, if two functions are inverses of each other	

SESSION 2

Specific Outcome 7: Demonstrate an understanding of logarithms.	
Achievement Indicator:	**Example:**
7.1Explain the relationship between logarithms and exponents.	
7.2Express a logarithmic expression as an exponential expression and vice versa.	
7.3Determine, without technology, the exact value of a logarithm, such as . $\log_2 8$	
7.4Estimate the value of a logarithm, using benchmarks, and explain the reasoning; e.g., since $\log_2 8 = 3$ and $\log_2 16 = 4$, $\log_2 9$is approximately equal to 3.1.	
Specific Outcome 8: Demonstrate an understanding of the product, quotient and power laws of logarithms.	
Achievement Indicator:	**Example:**
8.1Develop and generalize the laws for logarithms, using numeric examples and exponent laws.	

8.2Derive each law of logarithms.	
8.3Determine, using the laws of logarithms, an equivalent expression for a logarithmic expression.	
8.4Determine, with technology, the approximate value of a logarithmic expression, such as $\log_2 9$	
Specific Outcome 9: Graph and analyze exponential and logarithmic functions.	
Achievement Indicator:	**Example:**
9.1Sketch, with or without technology, a graph of an exponential function of the form *y* =a^x, *a* > 0.	
9.2Identify the characteristics of the graph of an exponential function of the form *y* = *a*^*x*, *a* > 0, including the domain, range, horizontal asymptote and intercepts, and explain the significance of the horizontal asymptote.	

9.3Sketch the graph of an exponential function by applying a set of transformations to the graph of *y* =a^x, *a* > 0, and state the characteristics of the graph.	
9.4Sketch, with or without technology, the graph of a logarithmic function of the form $y = \log_b x$, $b > 1$.	
9.5Identify the characteristics of the graph of a logarithmic function of the form $y = \log_b x$, $b > 1$, including the domain, range, vertical asymptote and intercepts, and explain the significance of the vertical asymptote.	
9.6Sketch the graph of a logarithmic function by applying a set of transformations to the graph of $y = \log_b x$, $b > 1$, and state the characteristics of the graph.	
9.7Demonstrate, graphically, that a logarithmic function and an exponential function with the same base are inverses of each other.	

Specific Outcome 10: Solve problems that involve exponential and logarithmic equations.	
Achievement Indicator:	**Example:**
10.1Determine the solution of an exponential equation in which the bases are powers of one another.	
10.2Determine the solution of an exponential equation in which the bases are not powers of one another, using a variety of strategies.	
10.3Determine the solution of a logarithmic equation, and verify the solution.	
10.4Explain why a value obtained in solving a logarithmic equation may be extraneous.	
10.5Solve a problem that involves exponential growth or decay.	

10.6Solve a problem that involves the application of exponential equations to loans, mortgages and investments.	
10.7Solve a problem that involves logarithmic scales, such as the Richter scale and the pH scale.	
10.8Solve a problem by modelling a situation with an exponential or a logarithmic equation.	
Specific Outcome 11: Demonstrate an understanding of factoring polynomials of degree greater than 2 (limited to polynomials of degree $\leq$ 5 with integral coefficients).	
Achievement Indicator:	**Example:**
11.1Explain how long division of a polynomial expression by a binomial expression of the form $x - a$, $a \in I$, is related to synthetic division.	
11.2Divide a polynomial expression by a binomial expression of the form $x - a$, $a \in I$, using long division or synthetic division.	

11.3Explain the relationship between the linear factors of a polynomial expression and the zeros of the corresponding polynomial function	
11.4Explain the relationship between the remainder when a polynomial expression is divided by $x - a$, $a \in I$, and the value of the polynomial expression at $x = a$ (remainder theorem).	
11.5Explain and apply the factor theorem to express a polynomial expression as a product of factors.	

Specific Outcome 12: Graph and analyze polynomial functions (limited to polynomial functions of degree ≤ 5).

Achievement Indicator:	Example:
12.1Identify the polynomial functions in a set of functions, and explain the reasoning.	
12.2Explain the role of the constant term and leading coefficient in the equation of a polynomial function with respect to the graph of the function.	

12.3Generalize rules for graphing polynomial functions of odd or even degree.	
12.4Explain the relationship between: • the zeros of a polynomial function • the roots of the corresponding polynomial equation • the *x*-intercepts of the graph of the polynomial function.	
12.5Explain how the multiplicity of a zero of a polynomial function affects the graph.	
12.6Sketch, with or without technology, the graph of a polynomial function.	
12.6Solve a problem by modelling a given situation with a polynomial function and analyzing the graph of the function.	

SESSION 3

Specific Outcome 13: Graph and analyze radical functions (limited to functions involving one radical).	
Achievement Indicator:	**Example:**
13.1 Sketch the graph of the function $y = \sqrt{x}$, using a table of values, and state the domain and range.	
13.2 Sketch the graph of the function $y - k = a\sqrt{b(x-h)}$ by applying transformations to the graph of the function $y = \sqrt{x}$, and state the domain and range.	
13.3 Sketch the graph of the function $y = \sqrt{f(x)}$, given the graph of the function $y = f(x)$, and explain the strategies used.	
13.4 Compare the domain and range of the function $y = \sqrt{f(x)}$, to the domain and range of the function $y = f(x)$, and explain why the domains and ranges may differ.	

13.5 Describe the relationship between the roots of a radical equation and the *x*-intercepts of the graph of the corresponding radical function.	
13.6 Determine, graphically, an approximate solution of a radical equation.	
Specific Outcome 14: Graph and analyze rational functions (limited to numerators and denominators that are monomials, binomials or trinomials).	
Achievement Indicator:	**Example:**
14.1 Graph, with or without technology, a rational function.	
14.2 Analyze the graphs of a set of rational functions to identify common characteristics.	
14.3 Explain the behaviour of the graph of a rational function for values of the variable near a non-permissible value.	

14.4Determine if the graph of a rational function will have an asymptote or a hole for a non-permissible value.	
14.5Match a set of rational functions to their graphs, and explain the reasoning.	
14.6Describe the relationship between the roots of a rational equation and the *x*-intercepts of the graph of the corresponding rational function.	
14.7Determine, graphically, an approximate solution of a rational equation.	

SESSION 4

Topic 3: Permutations, Combinations and Binomial Theorem	
General Outcome: Develop algebraic and numeric reasoning that involves combinatorics.	
Specific Outcome 1: Apply the fundamental counting principle to solve problems.	
Achievement Indicator:	**Example:**
1.1Count the total number of possible choices that can be made, using graphic organizers such as lists and tree diagrams.	
1.2Explain, using examples, why the total number of possible choices is found by multiplying rather than adding the number of ways the individual choices can be made.	
1.3Solve a simple counting problem by applying the fundamental counting principle.	
Specific Outcome 2: Determine the number of permutations of *n* elements taken *r* at a time to solve problems.	
Achievement Indicator:	**Example:**
2.1Count, using graphic organizers such as lists and tree diagrams, the number of ways of arranging the elements of a set in a row.	

2.2Determine, in factorial notation, the number of permutations of *n* different elements taken *n* at a time to solve a problem.	
2.3Determine, using a variety of strategies, the number of permutations of *n* different elements taken *r* at a time to solve a problem.	
2.4Explain why *n* must be greater than or equal to *r* in the notation $_nP_r$.	
2.5Solve an equation that involves $_nP_r$ notation, such as $_nP_2 = 30$	
2.6Explain, using examples, the effect on the total number of permutations when two or more elements are identical.	

Specific Outcome 3: Determine the number of combinations of *n* different elements taken *r* at a time to solve problems.	
Achievement Indicators:	**Example:**
3.1Explain, using examples, the difference between a permutation and a combination.	
3.2Determine the number of ways that a subset of *k* elements can be selected from a set of *n* different elements.	
3.3Determine the number of combinations of *n* different elements taken *r* at a time to solve a problem.	
3.4Explain why *n* must be greater than or equal to *r* in the notation $_nC_r$	
3.5Explain, using examples, why $_nC_r = {}_nC_{n-r}$	

3.6Solve an equation that involves ${}_nC_r$ such as ${}_nC_2 = 15$	
Specific Outcome 4: Expand powers of a binomial in a variety of ways, including using the binomial theorem (restricted to exponents that are natural numbers).	
Achievement Indicators:	**Example:**
4.1Explain the patterns found in the expanded form of $(x + y)^n$, $n \leq 4$ and $n \in N$, by multiplying n factors of $(x + y)$.	
4.2Explain how to determine the subsequent row in Pascal's triangle, given any row.	
4.3Relate the coefficients of the terms in the expansion of $(x + y)^n$ to the $(n + 1)$ row in Pascal's triangle.	
4.4Explain, using examples, how the coefficients of the terms in the expansion of $(x + y)^n$ are determined by combinations.	

4.5Expand, using the binomial theorem, $(x + y)^n$.	
4.6Determine a specific term in the expansion of $(x + y)^n$.	

SESSION 5

Topic 1: Trigonometry	
General Outcome: Develop trigonometric reasoning.	
Specific Outcome 1: Demonstrate an understanding of angles in standard position, expressed in degrees and radians.	
Achievement Indicators:	**Example:**
1.1Sketch, in standard position, an angle (positive or negative) when the measure is given in degrees.	
1.2Describe the relationship among different systems of angle measurement, with emphasis on radians and degrees.	
1.3Sketch, in standard position, an angle with a measure of 1 radian.	
1.4Sketch, in standard position, an angle with a measure expressed in the form $k\pi$ radians, where $k \in Q$.	

1.5Express the measure of an angle in radians (exact value or decimal approximation), given its measure in degrees.	
1.6Express the measure of an angle in degrees, given its measure in radians (exact value or decimal approximation).	
1.7Determine the measures, in degrees or radians, of all angles in a given domain that are coterminal with a given angle in standard position.	
1.8Determine the general form of the measures, in degrees or radians, of all angles that are coterminal with a given angle in standard position.	
1.9Explain the relationship between the radian measure of an angle in standard position and the length of the arc cut on a circle of radius *r*, and solve problems based upon that relationship.	

Specific Outcome 2: Develop and apply the equation of the unit circle.	
Achievement Indicators:	**Example:**
2.1Derive the equation of the unit circle from the Pythagorean theorem.	
2.2Describe the six trigonometric ratios, using a point P (*x*, *y*) that is the intersection of the terminal arm of an angle and the unit circle.	
2.3Generalize the equation of a circle with centre (0, 0) and radius *r*.	
Specific Outcome 3: Solve problems, using the six trigonometric ratios for angles expressed in radians and degrees.	
Achievement Indicator:	**Example:**
3.1Determine, with technology, the approximate value of a trigonometric ratio for any angle with a measure expressed in either degrees or radians.	

3.2Determine, using a unit circle or reference triangle, the exact value of a trigonometric ratio for angles expressed in degrees that are multiples of 0º, 30º, 45º, 60º or 90º, or for angles expressed in radians that are multiples of 0, π/6, π/4, π/3, or π/2 , and explain the strategy.	
3.3Explain how to determine the exact values of the six trigonometric ratios, given the coordinates of a point on the terminal arm of an angle in standard position.	
3.4Determine, with or without technology, the measures, in degrees or radians, of the angles in a specified domain, given the value of a trigonometric ratio.	
3.5Determine the measures of the angles in a specified domain in degrees or radians, given a point on the terminal arm of an angle in standard position.	
3.6Determine the exact values of the other trigonometric ratios, given the value of one trigonometric ratio in a specified domain.	

3.7Sketch a diagram to represent a problem that involves trigonometric ratios.	
3.8Solve a problem, using trigonometric ratios.	
Specific Outcome 4: Graph and analyze the trigonometric functions sine, cosine and tangent to solve problems.	
Achievement Indicator:	**Example:**
4.1 Sketch, with or without technology, the graph of y = sin x, y = cos x or y = tan x.	
4.2 Determine the characteristics (amplitude, asymptotes, domain, period, range and zeros) of the graph of y = sin x, y = cos x or y = tan x.	
4.3 Determine how varying the value of a affects the graphs of y = a sin x and y = a cos x.	

4.4 Determine how varying the value of d affects the graphs of $y = \sin x + d$ and $y = \cos x + d$.	
4.5 Determine how varying the value of c affects the graphs of $y = \sin (x + c)$ and $y = \cos (x + c)$.	
4.6 Determine how varying the value of b affects the graphs of $y = \sin bx$ and $y = \cos bx$.	
4.7 Sketch, without technology, graphs of the form $y = a \sin b(x - c) + d$ or $y = a \cos b(x - c) + d$, using transformations, and explain the strategies.	
4.8 Determine the characteristics (amplitude, asymptotes, domain, period, phase shift, range and zeros) of the graph of a trigonometric function of the form $y = a \sin b(x - c) + d$ or $y = a \cos b(x - c) + d$.	

4.9 Determine the values of *a*, *b*, *c* and *d* for functions of the form *y* = *a* sin *b*(*x* − *c*) + *d* or *y* = *a* cos *b*(*x* − *c*) + *d* that correspond to a given graph, and write the equation of the function.	
4.10Determine a trigonometric function that models a situation to solve a problem.	
4.11Explain how the characteristics of the graph of a trigonometric function relate to the conditions in a problem situation.	
4.12 Solve a problem by analyzing the graph of a trigonometric function.	

SESSION 6

Specific Outcome 5: Solve, algebraically and graphically, first and second degree trigonometric equations with the domain expressed in degrees and radians.	
Achievement Indicator:	**Example:**
5.1 Verify, with or without technology, that a given value is a solution to a trigonometric equation.	
5.2 Determine, algebraically, the solution of a trigonometric equation, stating the solution in exact form when possible.	
5.3 Determine, using technology, the approximate solution of a trigonometric equation in a restricted domain.	
5.4 Relate the general solution of a trigonometric equation to the zeros of the corresponding trigonometric function (restricted to sine and cosine functions).	
5.5 Determine, using technology, the general solution of a given trigonometric equation.	

5.6 Identify and correct errors in a solution for a trigonometric equation.	
Specific Outcome 6: Prove trigonometric identities, using: · reciprocal identities · quotient identities · Pythagorean identities · sum or difference identities (restricted to sine, cosine and tangent) · double-angle identities (restricted to sine, cosine and tangent).	
Achievement Indicator:	**Example:**
6.1 Explain the difference between a trigonometric identity and a trigonometric equation.	
6.2 Verify a trigonometric identity numerically for a given value in either degrees or radians.	
6.3 Explain why verifying that the two sides of a trigonometric identity are equal for given values is insufficient to conclude that the identity is valid.	
6.4 Determine, graphically, the potential validity of a trigonometric identity, using technology.	

6.5 Determine the non-permissible values of a trigonometric identity.	
6.6 Prove, algebraically, that a trigonometric identity is valid.	
6.7 Determine, using the sum, difference and double-angle identities, the exact value of a trigonometric ratio.	